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THE SPECTRAL DENSITY METHOD APPLIED TO THE HOT QUANTUM FIELD THEORIES

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ABSTRACT

The spectral density method being applied to the quantum field theory at finite temperature is revived and its possibilities are briefly discussed.

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1 Introduction

The spectral density method proposed for the first time in 1968 [1] is surveyed and we find it to be useful for studying the hot gauge theory and other relativistic models. This method is shown through two models but one can see that its main content keeps the same as was done previously (see e.g. Ref.[2] for details). We demonstrate that its simplest approximation is very convenient to fix the nontrivial vacuum and to build a reliable calculational scheme on this basis. To date, the separate fragments of this method have been already used by many authors for different applications (see e.g. [3,4] and [5]) although its complete formalism remains unknown. The goal of this note is to represent this formalism and to show that no peculiarities arise when its formulae are applied to the relativistic models which are intensively studied now but (unfortunately) they were not considered in [2]. Below, this method is briefly introduced in accordance with the paper [2] and its simplest approximation is discussed.

2 The general formalism

The spectral density is determined as a standard statistical average of a non-equal-time commutator or anticommutator ($\eta = \mp 1$) of some operators A and B:

$$\Lambda(\omega) = \eta \langle [A; B(\tau)]_\eta \rangle_\omega = \int d\tau \exp(i\omega\tau) (\eta [A; B(\tau)]_\eta) \quad (1)$$

and should be found through the exact set of integral relations

$$\int \frac{d\omega}{2\pi} \omega^m (\eta \langle [A; B(\tau)]_\eta \rangle_\omega) = \langle [[H[H\dots[H; A]_{-}\dots]; B]_\eta \rangle \quad (2)$$

which are a direct consequence of the equation of motion for the operator $B(\tau)$

$$i \frac{\partial}{\partial \tau} B(\tau) = [B(\tau); H(\tau)]_- \quad (3)$$

This is the essence of the spectral density method and the connection with the standard Green function formalism is established through the following

formula

$$\mathcal{D}(\omega) = \int \frac{dz}{2\pi} \frac{\Lambda(z)}{z - \omega} \quad (4)$$

where the standard analytical conditions should be taken into account. The iteration ansatz within Eq.(2) and another useful formulae can be found in [2] but it is not our goal to rewrite them again. In what follows we shall try to demonstrate this method and discuss its possibilities.

3 U(1)-scalar field

This is the simplest case for testing our method. The Hamiltonian is defined to be

$$H = \frac{1}{2V} \sum_{\mathbf{p}} \left[\pi_{\mathbf{p}} \pi_{-\mathbf{p}} + (\mathbf{p}^2 + m^2) \varphi_{\mathbf{p}} \varphi_{-\mathbf{p}} \right] \quad (5)$$

where the system is put into the three dimensional black box V with the periodic boundary conditions. This is a useful way to simplify the standard commutation relations and to avoid the needless difficulties. Now all moments for the standard spectral density

$$\Lambda(\mathbf{p}, \omega) = - \langle [\varphi_{\mathbf{p}}(0); \varphi_{\mathbf{p}}(\tau)]_- \rangle_{\omega} \quad (6)$$

are easily calculated and found to be

$$\int \frac{d\omega}{2\pi} \omega \Lambda(\mathbf{p}, \omega) = 1, \quad \int \frac{d\omega}{2\pi} \omega^{2n+1} \Lambda(\mathbf{p}, \omega) = (\mathbf{p}^2 + m^2)^n \quad (7)$$

where $n = 1, 2$ and so on. Only the odd moments are not equal zero and this is the general situation for all relativistic systems. The exact spectral density has the form

$$\Lambda(\mathbf{p}, \omega) = \frac{\pi}{\omega(\mathbf{p})} [\delta(\omega - \omega(\mathbf{p})) - \delta(\omega + \omega(\mathbf{p}))] \quad (8)$$

and all moments will be satisfied if $\omega(\mathbf{p}) = \sqrt{\mathbf{p}^2 + m^2}$.

4 SU(N)-gauge theory

The first main question is to choose the gauge and to define the Hamiltonian in the most simple way. This situation is typical for any calculations within a gauge theory and should be solved at the beginning in the standard manner. To this end the temporal axial gauge is fixed where the Hamiltonian is known to be

$$H = \frac{1}{2}\pi_i^a\pi_i^a + \frac{1}{4}B_i^aB_i^a, \quad B_i^a = \partial_j V_l^a - \partial_l V_j^a + gf^{abc}V_j^bV_l^c \quad (9)$$

and the usual gluon propagator has the form

$$\mathcal{D}_{ij}(p_4, \mathbf{p}) = \frac{1}{(p_4^2 + \mathbf{p}^2) + A(p)} \left(\delta_{ij} - \frac{p_i p_j}{\mathbf{p}^2} \right) + \frac{1}{p_4^2 + (p_4^2/\mathbf{p}^2)\Pi_{44}(p)} \frac{p_i p_j}{\mathbf{p}^2} \quad (10)$$

Both scalar functions $A(p_4, \mathbf{p})$ and $\Pi_{44}(p_4, \mathbf{p})$ have the completely different limits in accordance with the ratio $p_4/|\mathbf{p}|$ and here only the so-called "plasmon limit" (but not the infrared one) is essential. This limit for the small momenta was calculated many years ago [6] and known to be

$$A(p_4, \mathbf{p}) = m^2 \left(1 - \frac{\mathbf{p}^2}{5p_4} \right), \quad \frac{p_4^2}{\mathbf{p}^2} \Pi_{44}(p_4, \mathbf{p}^2) = m^2 \left(1 - \frac{3\mathbf{p}^2}{5p_4^2} \right) \quad (11)$$

where $m^2 = g^2 T^2 N/9$. The standard spectra

$$\omega_t^2(\mathbf{p}) = m^2 + \frac{6}{5}\mathbf{p}^2, \quad \omega_l^2(\mathbf{p}) = m^2 + \frac{3}{5}\mathbf{p}^2 \quad (12)$$

result from the additional iteration within (10) when p_4^2 replaces to $(-m^2)$ for Eq.(11). This means that the simplest approximation within the spectral density method should reproduce another spectra which ignore the last iteration but (as we shall see) give the qualitatively correct result.

The correspondent spectral density is defined to be

$$\Lambda_{i,j}(\mathbf{p}, \omega) = - < [V_{i,\mathbf{p}}(0); V_{j,\mathbf{p}}(\tau)]_- > \omega \quad (13)$$

and has a standard tensor structure

$$\Lambda_{i,j}(\mathbf{p}, \omega) = \left(\delta_{ij} - \frac{p_i p_j}{\mathbf{p}^2} \right) \Lambda_t(\mathbf{p}, \omega) + \frac{p_i p_j}{\mathbf{p}^2} \Lambda_l(\mathbf{p}, \omega) \quad (14)$$

Its first nonzero moments are easily calculated

$$\int \frac{d\omega}{2\pi} \omega \Lambda_{i,j}(\mathbf{p}, \omega) = \delta_{i,j}, \quad \int \frac{d\omega}{2\pi} \omega^3 \Lambda_{i,j}(\mathbf{p}, \omega) = [(\mathbf{p}^2 + m^2)\delta_{i,j} - p_i p_j] \quad (15)$$

and allow to find all parameters if the spectral density has the standard form

$$\Lambda_n(\mathbf{p}, \omega) = \frac{\pi}{\omega_n(\mathbf{p})} [\delta(\omega - \omega_n(\mathbf{p})) - \delta(\omega + \omega_n(\mathbf{p}))] \quad (16)$$

where $n = t, l$. The result is

$$\omega_t^2(\mathbf{p}) = \mathbf{p}^2 + m^2, \quad \omega_l^2(\mathbf{p}) = m^2 \quad (17)$$

and one can see that this is qualitatively the same as Eq.(12). But within the spectral density method we can find a selfconsistent equation for m^2 which is obtained in (15) to be

$$\delta_{i,j} m^2 \delta^{a,b} = -\frac{1}{2} \Gamma_{ijmn}^{abcd} \int \frac{d^3 \mathbf{p}}{(2\pi)^3} < V_{m,\mathbf{p}}^c V_{n,-\mathbf{p}}^d > \quad (18)$$

after the third moment has been calculated. Using the known formula from Ref.[2]

$$< A B > -\theta(-\eta) < A > < B > = \int \frac{d\omega}{2\pi} \frac{(\eta < [A, B(\tau)]_\eta > \omega)}{\exp(\beta\omega) + \eta} \quad (19)$$

and the standard dimensional regularization one find this equation to be

$$m^2 = \frac{4g^2 N}{3} \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \frac{1}{\sqrt{m^2 + \mathbf{p}^2}} \frac{1}{\exp[\beta\sqrt{m^2 + \mathbf{p}^2}] - 1} \quad (20)$$

which reproduces the known value $m^2 = g^2 T^2 N/9$ in the high temperature limit. The more complicated approximations for $\Lambda_{i,j}(\omega)$ are also available (see Ref.[2]) to obtain Eq.(12) and other peculiarities of the SU(N)-gauge theory but it is not subject of this note.

5 Conclusion

Here we demonstrate that the spectral density method (being very simple) is able to give the qualitatively correct spectra (and other quantities) for any quantum system and can be used to build a reliable calculational scheme beyond the standard perturbative expansion.

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7 References

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